second being $(f_1+f_2+f_2/m)$, and that between the second and third $(f_2+f_3+mf_2)$.

If the lenses are nearly but not quite in the afocal position, greater power and a wider field may be obtained, but it is at the expense of the penetration, which may, however, with advantage be limited to the thickness of the object. The instrument offers great advantages for artistic purposes, but lenses or mirrors of specially wide angle are needed for the farther development of the invention.

The optical conditions of a system of two thin lenses at varying distance apart are shown by diagrams.

In Diagram I the u and v of the formula employed are set off as abscissæ and ordinates, and the curves (which are rectangular hyperbolas) drawn for several values of H. In the afocal position of the lenses the curve degrades into a line which is a tangent to all the hyperbolas at the point (f_1, f_2). The locus of vertices and locus of centres of these curves being straight lines, and the hyperbolas all touching the point (f_1, f_2), it is shown that the principal foci, principal points, and equivalent focal length for any given position of the lenses can be found by rule and compasses, without drawing the curve.

In Diagram II the actual position of the lenses, their principal foci, separate and combined, and the principal points, positive and negative (answering to the vertices of the curves in Diagram I), are plotted down as abscissæ, the values of H on an enlarged scale being taken as ordinates.

Diagram III shows the same for two lenses of equal focal length.

Comparison of these two diagrams suggests the employment of the term "Pseudo-Principal Points" for those positions at which the magnitude of the image is in the constant ratio f_2/f_1 to that of the object for every value of H, inasmuch as the distance from these to the principal points gives the measure of the "penetration" of the system.

II. "On the Thermodynamic Properties of Substances whose Intrinsic Equation is a Linear Function of the Pressure and Temperature." By Professor GEORGE F. FITZGERALD, M.A., F.R.S. Received January 11, 1887.

Professor Ramsay has communicated to me that he and Mr. Young have found that within wide limits several substances in the liquid and gaseous states have the following relation connecting their pressure (p), temperature (T), and specific volume (v),

$$p = a\mathrm{T} + b,$$

where a and b are functions of v only.

Now in this case the following are the forms that the thermodynamic equations assume. T is temperature, and ϕ is entropy, and c and e are functions to be investigated, c being $= d\mathrm{I}/d\mathrm{T}$, where I is the internal energy, and $e = d\mathrm{I}/dv$.

Then

$$\mathrm{T}\, d\phi = c\, d\mathrm{T} + (e + p)dv.$$

From this, as $de/d\mathrm{T} = dc/dv$, we have $dp/d\mathrm{T} = e+p/\mathrm{T}$.

But $dp/d\mathrm{T} = a$ by the intrinsic equation, and is a function of v only, $\therefore e = -b$, which is a function of v only, $\therefore de/d\mathrm{T} = 0$, $\therefore de/dv = 0$, $\therefore$ is c a function of T only, and $d\mathrm{I} = c\, d\mathrm{T} + e\, dv$ gives—

$$\mathrm{I} = \int c\, d\mathrm{T} + \int e\, dv = \gamma + \lambda,$$

where γ is a function of temperature only, and λ a function of volume only.

Similarly,

$$\mathrm{T}\, d\phi = c\, dt + a\mathrm{T}\, dv;$$

$$\therefore\; d\phi = \frac{c}{\mathrm{T}} \,.\, dt + a\, dv,$$

$$\therefore\; \phi = \int \frac{c\, dt}{\mathrm{T}} + \int a\, dv,$$

$$\phi = \Gamma + \alpha,$$

where Γ is a function of temperature and α of volume only.

Hence we see that c, the specific heat at a constant volume, is a function of the temperature only, and the internal energy and the entropy can be expressed as the sums of two functions, one a function of the temperature only, and the other of the volume only.

For the specific heat at constant pressure we have—

$$\mathrm{C} = c + (e+p)\frac{dv}{dt}$$

$$= c + a\mathrm{T} \,.\, \frac{-a}{\mathrm{T}a' + b'},$$

where $\quad a' = da/dv \quad$ and $\quad b' = db/dv,$

$$\therefore\; \mathrm{C} - c = -\frac{\mathrm{T}a^2}{\mathrm{T}a' + b'}.$$

In the case of the particular values of a and b that Professor Ramsay has suggested to me, when the intrinsic equation assumes the form—

$$p = \frac{\mathrm{RT}}{v - v_0} - \frac{\mu}{v^n},$$

and when consequently

$$a = \frac{R}{v - v_0}, \qquad b = -\frac{\mu}{v^n},$$

we have

$$e = \mu v^{-n},$$

and

$$I = \gamma - \frac{(n-1)\mu}{v^{n-1}},$$

$$\phi = \Gamma + R \log (v - v_0),$$

$$C - c = \frac{TR^2}{TR + n\mu(v - v_0)^2 v^{-n-1}}.$$

It would be most important if by some method, Kœnig's for instance, or by inserting a small microphone into a tube, the velocity of sound in substances in various states could be accurately determined, as that would enable us to determine C and c separately.

III. "On the Morphology of Birds." By Professor W. K. Parker, F.R.S. Received January 13, 1887.

(Abstract.)

Introductory Remarks.

During the time that the special study of the development of the skull has occupied my attention, the rest of the skeleton has been neglected; it has, however, had its cultivators in no small number.

In a limited degree the skeleton has been worked out by me;—for instance, the shoulder-girdle and sternum in the Vertebrata generally; in birds, the whole skeleton did at one time—a quarter of a century ago—take up much of my thought.

The *development* of the skeleton, generally in this Class, is a subject of great interest, and I am anxious to catch up all the scattered results that lie before me, of the excellent but extremely limited labours of other biologists.

I did begin the study of the development of the limbs, sternum, pelvis, and spine, in 1842, and some of the results will be brought forward in the present paper.

This will be, I trust, but the first-fruits of my most recent work; for, during the long years that have elapsed since this research was fairly begun, I have lost no opportunity of laying up in store embryos and young of birds of many kinds. These stores, if well worked out, will yield a series of papers like the one now offered to the Society.*

* Although I have for many years past kept a register of the presents of